Final Exam in Modeling and Simulation (CS-433)

Course Documents Authorized
Duration: 120 minutes

Questions are generally independent. You can respond to any question before the other.

- يمكنك الإجابة على أي سؤال قبل أي سؤال آخر لأن الأسئلة عموما غير مترابطة.
- الرجاء إعادة أوراق الإجابة فقط، أوراق الأسئلة لا تعاد إلى الأستاذ.

Grading

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الإسم الثالثي: ............................................
المستوى ........................................................
النتيجة: 40/40
Exercise 1. General Questions (5 points)  

You can have more than one correct answer in each question  
Each correct answer gives +0.5 and each wrong answer gives -0.5.  
There are eight correct answers. There is at least one correct answer in each question.

1- What is a state of the system in the discrete-event simulation?
   A It is a variable that represents the simulation time
   B It is variable that represents an event in the system
   C It is a variable that represents the behavior of the system
   D It is variable that represents describe the changes that occur in the system

2- The confidence interval
   A becomes larger when the confidence level decreases
   B becomes larger when the confidence level increases
   C becomes larger when the variance of the sample increases
   D becomes larger when the sample size increases

3- In discrete-event simulations
   A Events are stored in a future event list in increasing order of their time stamps
   B The simulation time progresses at regular time steps
   C Events are stored in a future event list in decreasing order of their time stamps
   D The simulation time progresses at according to the event time stamps

4- In Continuous Markov Chains
   A The transition probability to the same state is always equal to zero
   B The steady state probability can be different from zero
   C The CDF of the state holding time is $F(x) = 1 - \exp(-2x)$, $-qii$ is the output rate
   D The output transition rate is equal to the sum of input transition rates.
Exercise 2. Queueing Theory (10 points) (20 minutes)

Consider an M/M/1 queuing system with an arrival rate $\lambda = 0.4$ and service rate $\mu = 0.5$.

1. Compute the system load and tell if the system stable or not? (1 point)
   \[
   \rho = \frac{\lambda}{\mu} = \frac{0.4}{0.5} = 0.8
   \]
   The system is stable because $\rho < 1$

2. Compute the average number of customers in the system? (1 point)
   \[
   E[X] = \frac{\rho}{1 - \rho} = \frac{0.8}{1 - 0.8} = 4
   \]
   We made a simulation run and we find that the average number of customers is equal to 6.32.

3. What are the TWO possible reasons that explain the difference between simulation and analytical results? (3 points)
   - The simulation time may be too short, and thus did not reach the steady state
   - The random number generator may be bad

4. List TWO advantages of simulation models as compared to analytical models (2 points).
   - Simulation can be used for highly complex system where analytical models are not possible.
   - Simulations are more flexible than mathematical modeling and have fewer assumptions.

Exercise 3. Queueing Theory (10 points) (20 minutes)

Consider a computer system with two processors and one infinite queue. Tasks arrive to the processors according to a Poisson process with a rate $\lambda = 20$ tasks/sec and the each server has a processing speed of $C = 2$ Mbits/sec. The service time is exponentially distributed with a mean $\frac{1}{\mu} = 50$ ms.

1. What is the service rate of each processor expressed in tasks/sec.
   The service rate $\mu = \frac{1}{50 \text{ ms}} = 20$ tasks/sec

2. Compute the average task size (L) in Kbits? (1 point)
   (assume that 1 Mbits = 1000 Kbits)
   \[
   \text{The average size of the task is } L = \frac{2000 \text{ Kbits/sec}}{20 \text{ tasks/sec}} = 100 \text{ Kbits}
   \]

3. Determine the queue model of the computer system and compute the expected queue length of the system? (2 points)
This is an M/M/2 queue model as there are two servers with the same rate, an infinite queue, a Poisson arrival and exponential service time.

The expected queue length is:

\[ E[X] = 2\rho + \frac{(2\rho)^2}{2!} \frac{\rho}{(1-\rho)^2} \pi_0 \]

\[ \rho = \frac{\lambda}{2\mu} = \frac{20}{2 \times 20} = 0.5 \]

We need to compute

\[ \pi_0 = \frac{1}{\left[1 + (2 \cdot \rho) + \frac{4 \cdot \rho^2}{2 \cdot (1-\rho)}\right]} = \frac{1}{\left[1 + (2 \cdot 0.5) + \frac{4 \cdot (0.5)^2}{2 \cdot (1-0.5)}\right]} = \frac{1}{1 + 1 + \frac{1}{3}} = \frac{1}{\frac{3}{3}} = 1 \]

Thus,

\[ E[X] = 2\rho + \frac{(2\rho)^2}{2!} \frac{\rho}{(1-\rho)^2} \pi_0 = 1 + 1 \frac{0.5}{2} + 0.25 \frac{1}{3} = 1 + \frac{1}{3} = \frac{4}{3} = 1.333 \text{ tasks} \]

4. If an M/M/1 queue model was used for the computer system with an arrival rate \( \lambda = 20 \text{ tasks/sec} \) and one processor with a service rate \( \mu = 40 \text{ tasks/sec} \).

Decide if the M/M/1 queue model will provide faster response time or not as compared to the previous queue model of the computer system?

(2 points)

The response time for the M/M/1 queue model is:

\[ E[S_{M/M/1}] = \frac{1}{\mu-\lambda} = \frac{1}{40-20} = \frac{1}{20} = 0.05 \text{ sec} \]

The response time in the M/M/2 queue is

\[ E[S_{M/M/2}] = \frac{E[X_{M/M/2}]}{\lambda} = \frac{4 \text{ tasks}}{40 - 20} = \frac{4}{3 \times 20} = 0.066 \text{ sec} \]

Thus, the M/M/1 queue model would be faster than the M/M/2 queue model.
Exercise 4. Simulation (16 points) (75 minutes)

Consider the network simulation problem as shown in the following figure. We propose to study the performance of a network router which has a buffer to store incoming packets from a video server and phone server in a local network and a constant transmission speed equal to $C = 1 \text{Mbit/sec}$.

![Network Simulation Model](image)

We assume that the router has an infinite buffer size. The first station sends video stream at a rate $\lambda_1 \text{ packets/sec}$ and the second station sends a voice stream at a rate $\lambda_2 \text{ packets/sec}$. We assume that both streams follow a Poisson Arrival Process and that the total arrival rate $\lambda = \lambda_1 + \lambda_2 = 55 \text{ packets/sec}$. All Packets have an exponentially distributed service time with the mean $1/\mu$.

We make two simulation runs with different simulation times, and we observe the results of waiting time. Table 1 presents the simulation and analytical results of the waiting time for this network.

<table>
<thead>
<tr>
<th>Rho</th>
<th>Waiting Time (ms) Analytical</th>
<th>Waiting Time (ms) Run#1</th>
<th>Waiting Time (ms) Run#2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.2020</td>
<td>0.2036</td>
<td>0.2027</td>
</tr>
<tr>
<td>0.2</td>
<td>0.9091</td>
<td>0.9004</td>
<td>0.9077</td>
</tr>
<tr>
<td>0.3</td>
<td>2.3377</td>
<td>2.3383</td>
<td>2.3459</td>
</tr>
<tr>
<td>0.4</td>
<td>4.8485</td>
<td>4.8241</td>
<td>4.8654</td>
</tr>
<tr>
<td>0.5</td>
<td>9.0909</td>
<td>8.8637</td>
<td>9.1200</td>
</tr>
<tr>
<td>0.6</td>
<td>16.3636</td>
<td>15.5304</td>
<td>16.3455</td>
</tr>
<tr>
<td>0.7</td>
<td>29.6970</td>
<td>27.5535</td>
<td>29.9064</td>
</tr>
<tr>
<td>0.8</td>
<td>58.1818</td>
<td>52.9243</td>
<td>59.3392</td>
</tr>
<tr>
<td>0.9</td>
<td>147.2727</td>
<td>123.8460</td>
<td>146.6010</td>
</tr>
</tbody>
</table>

The same results are also presented in Figure 1.

1. Did Simulation run#1 and Simulation run#2 reach the steady state? Explain (2 points).

Simulation Run# 2 has reached the steady state because all its values are close to the analytical results. However, Simulation run#1 did not reach the steady state because its values at high loads are far from the analytical results.
What are the maximum system load and the minimum service rate (packets/sec) such that the Queuing Time do not exceed 17 ms? (2 points)

According to the figure, the system load must not exceed 0.6. This means that the maximum service $\rho = \frac{\lambda}{\mu} < 0.6 \Rightarrow \frac{1}{\mu} < \frac{0.6}{\lambda} = 0.0109 \, ms$

Thus the minimum service rate is 91.67 packets/sec.

3. Compute the margin of error and the confidence interval of waiting time for $\rho = 0.9$ for both runs assuming the sample size and standard deviation presented in Table 2.

Table 2. Sample Size and Std Dev of the simulation runs

<table>
<thead>
<tr>
<th>Rho</th>
<th>Sample Size</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run#1</td>
<td>234</td>
<td>87.70</td>
</tr>
<tr>
<td>Run#2</td>
<td>3229</td>
<td>60.48</td>
</tr>
</tbody>
</table>

The margin of error in run#1 is: $\pm 1.96 \times \frac{87.70}{\sqrt{234}} = \pm 11.236$

The confidence interval for run #1 is then: $[123.84 - 11.236, 123.84 + 11.236] = [112.6091, 135.0829]$

The margin of error in run#1 is: $\pm 1.96 \times \frac{60.48}{\sqrt{3229}} = \pm 2.0860$

The confidence interval for run #1 is then: $[146.6 - 2.086, 146.6 + 2.086] = [144.5149, 148.6871]$
4. What do you conclude on the confidence on the results of simulation run #1 and simulation run #2?

It can be easily observed that run#2 provided more confidence on the results rather than run#1, because for the same confidence level, it has a shorter